~~Name~~ p. 1

Proving that a Quadrilateral is a Parallelogram

Lesson Objective IWBAT prove that a quadrilateral is or is not a parallelogram.

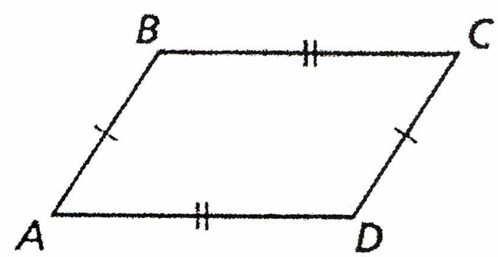
You can only prove that a quadrilateral is a parallelogram using its definition (below):

quad. w/ both pairs of opp. sides //

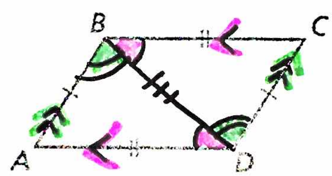
However, there are theorems that allow us to use other properties of quadrilaterals to conclude that it is a parallelograms. They are as follows:

Parallelogram Opposite Sides Converse

If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



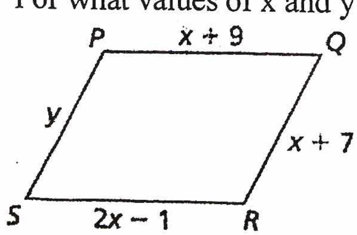
PROOF: **Given:** $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$
Prove: ABCD is a parallelogram



Statements	Reasons
① $\overline{AB} \cong \overline{CD}$, $\overline{BC} \cong \overline{DA}$	① given
② Draw $\overline{BD}$	② Thru 2 pts is 1 line
③ $\overline{BD} \cong \overline{DB}$	③ reflex. prop. $\cong$
④ $\triangle ABD \cong \triangle CDB$	④ SSS $\cong$
⑤ $\angle CBD \cong \angle ADB$, $\angle ABD \cong \angle CDB$	⑤ CPCTC
⑥ $\overline{BC} \parallel \overline{AD}$, $\overline{AB} \parallel \overline{CD}$	⑥ alt. int. ∠s $\cong \rightarrow \parallel$
⑦ ABCD is a //gram	⑦ both pairs opp. sides // → //gram

Example:

1. For what values of x and y is quadrilateral PQRS a parallelogram?



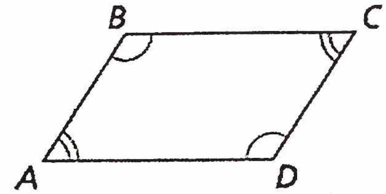
$x+9=2x-1$

$\boxed{x=10}$ $\boxed{y=17}$

Parallelogram Opposite Angles Converse

If both pairs of opposite angles of a quadrilateral are congruent,

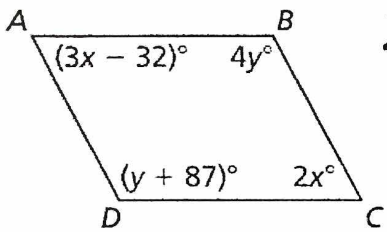
then the quadrilateral is a parallelogram.



PROOF: end of class (if time permits). Will be on Edline tonight if you are so curious to look it up ☺

Example:

2. For what values of x and y is quadrilateral ABCD a parallelogram?



$2x = 3x - 32$

$\boxed{x = 32}$

$4y = y + 87$

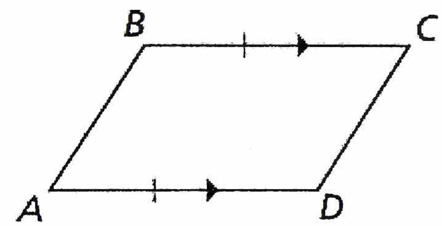
$3y = 87$

$\boxed{y = 29}$

Opposite Sides Parallel and Congruent Theorem

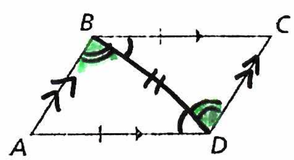
If one pair of opposite sides of a quadrilateral are congruent and parallel,

then the quadrilateral is a parallelogram.



PROOF: **Given:** $\overline{BC} \cong \overline{AD}$ and $\overline{BC} \parallel \overline{AD}$

Prove: ABCD is a parallelogram



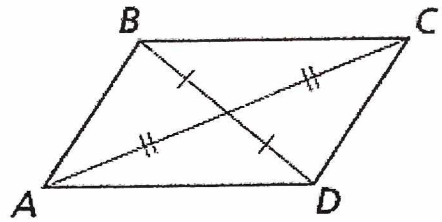
Remember, you can use any past theorems you proved to be true as reasons!!!!

Statements	Reasons
1 $\overline{AD} \cong \overline{BC}$, $\overline{BC} \parallel \overline{AD}$	1 given
2 Draw $\overline{BD}$	2 Thru 2 pts. is 1 line
3 $\overline{BD} \cong \overline{DB}$	3 reflex. prop. $\cong$
4 $\angle CBD \cong \angle ADB$	4 $\parallel \rightarrow$ alt. int. $\angle$s $\cong$
5 $\triangle CBD \cong \triangle ADB$	5 SAS $\cong$
6 $\angle ABD \cong \angle CDB$	6 CPCTC
7 $\overline{AB} \parallel \overline{CD}$	7 alt. int. $\angle$s $\cong \rightarrow \parallel$
8 ABCD is a $\parallel$-gram	8 both pairs opp. sides $\parallel$ $\rightarrow \parallel$ gram

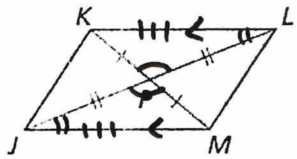
Parallelogram Diagonals Converse

If the diagonals of a quadrilateral bisect each other,

then the quadrilateral is a parallelogram.



PROOF: **Given**: Diagonals $\overline{JL}$ and $\overline{KM}$ bisect each other
Prove: JKLM is a parallelogram



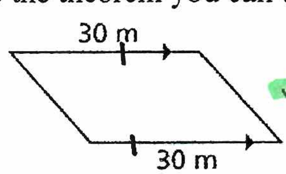
Remember, you can use any past theorems you proved to be true as reasons!!!!

Statements	Reasons
① $\overline{JL}, \overline{KM}$ bisect each other	① given
② $\overline{KP} \cong \overline{MP}$, $\overline{JP} \cong \overline{LP}$	② def. of seg. bisector
③ $\angle KPL \cong \angle MPJ$	③ vert. $\angle$s $\cong$
④ $\triangle KPL \cong \triangle MPJ$	④ SAS $\cong$
⑤ $\overline{KL} \cong \overline{MJ}$, $\angle PJM \cong \angle PLK$	⑤ CPCTC
⑥ $\overline{KL} \parallel \overline{MJ}$	⑥ alt. int. $\angle$s $\cong \rightarrow \parallel$
⑦ JKLM is a //gram	⑦ same pair opp. sides $\cong$ AND // $\rightarrow$ //gram

Example:

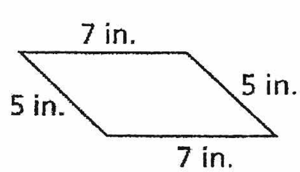
3. State the theorem you can use to show that the quadrilateral is a parallelogram.

a.



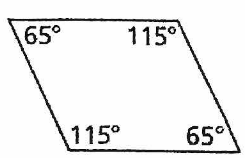
same pair opp. sides $\cong$ AND // $\rightarrow$ //gram

b.



both pairs opp sides $\cong \rightarrow$ //gram

c.



both pairs opp. $\angle$s $\cong \rightarrow$ //gram